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Note

An Example of a Nonpoised Interpolation Problem with a Constant Sign Determinant

In the work of S. Karlin and J. M. Karon [1] a perturbation technique is presented by which new collections of nonpoised interpolation problems can be generated out of known ones. The technique is based on the following theorem ([1], Theorem 2.1) proved there:

"*Let F be an incidence matrix for an $H - B$ polynomial interpolation problem which is not order-poised and which changes sign in any neighborhood of some zero of the determinant $K(F)$. Let E be any incidence matrix from which F may be obtained by coalescing some of the rows of E. Then E is not order-poised, and the determinant $K(E)$ changes sign at least one of its zeroes.*"

The requirement that $K(F)$ changes sign in any neighborhood of one of its zeroes is essential to the method of proof, but it is not commented on in [1] whether or not this requirement is essential to the validity of the theorem.

In the following we present an example of a non-order poised incidence matrix F from which an order poised matrix E_1 is obtained by perturbation. The determinant $K(F)$ is of constant sign near its zero, indicating that the above theorem is valid only in case there is a change of sign in a neighborhood of a zero of the determinant $K(F)$. Moreover, by different perturbations we can get from F a non-order poised matrix E_3 such that $K(E_3)$ is of constant sign, as well as a non-order poised matrix E_2 such that $K(E_2)$ changes sign in the neighborhood of its zero.

It should be mentioned that up to now no example of a non-order poised problem with a determinant of constant sign was known.

The matrices are:

$$F = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \qquad E_1 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix}$$

$$E_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{matrix} \qquad E_3 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix}$$

If we take $x_2 = 0$ it is easily seen that $K(F) \leqslant 0$ for all $x_1 < 0 < x_3$ and $K(F) = 0$ for $x_3 = -x_1$. A similar calculation shows that $K(E_1) \neq 0$ for all $x_1 < 0 < x_3 < x_4$ and yet F can be obtained from E_1 by coalecsing row 4 to row 3 ($x_4 \to x_3{}^+$). In order to verify the rest of the claims one can decompose E_2 and E_3 into irreducible matrices (see, for example [2]) and for the matrix E_2 use Theorem 2.2 of [1].

References

1. S. Karlin and J. M. Karon, Poised and nonpoised Hermite–Birkhoff interpolation, *Ind. Univ. Math. J.* **21** (1972), 1131–1170.
2. K. Atkinson and A. Sharma, A partial characterization of poised Hermite–Birkhoff interpolation problems, *SIAM J. Numer. Anal.* **6** (1969), 230–235.

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